Things to Know for the Geometry Regents Exam

ANGLES

Angles INSIDE Triangles: add to 180°

Angles INSIDE Quadrilaterals: add to 360°

Angles INSIDE any polygon with "*n*" sides: add to $180°(n-2)$

If polygon is regular (all sides and all angles are congruent) then

EACH angle inside the polygon measures: $\frac{180°(n-2)}{n}$

Angles OUTSIDE any polygon: add to $360°$ where each exterior angle in a *regular* polygon measures $\frac{360}{n}$

Polygons to Know:

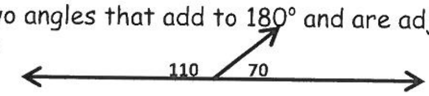
Name	Triangle	Quadrilateral	Pentagon	Hexagon	Octagon	Decagon
# of sides	3	4	5	6	8	10

Complementary Angles: two angles that add to 90°

Supplementary Angles: two angles that add to 180°

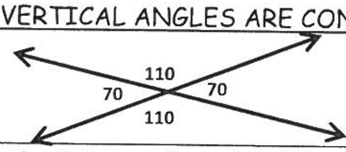
Linear Pair: two angles that add to 180° and are adjacent (form a line)

Ex)



Vertical Angles: (the angles opposite one another that are formed when two lines intersect) VERTICAL ANGLES ARE CONGRUENT.

Ex)



AREA

*Note: When finding the **Volume** of a solid, **"B"** stands for the **Area** of Base.*

Square: $A = s^2$ **Rectangle**: $A = LW$ **Triangle**: $A = \frac{1}{2}bh$

Circle: $A = \pi r^2$ **Trapezoid**: $A = \frac{1}{2}h(b_1 + b_2)$

COORDINATE GEOMETRY including CIRCLES

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of a Circle: where r = radius

centered at origin: $x^2 + y^2 = r^2$

centered at (h, k): $(x - h)^2 + (y - k)^2 = r^2$

Ex) What is the center and radius of this circle:

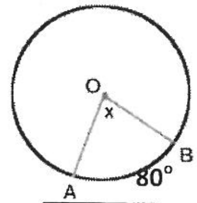
$(x - 3)^2 + (y + 5)^2 = 16$?

Center: (+3, -5)
Radius: $\sqrt{16} = 4$

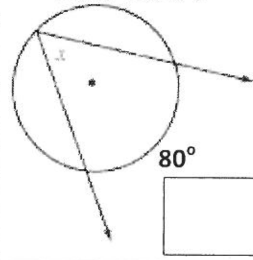
Notice: Change the signs of x and y to find center If no number is written (as in x^2), then use zero. Also, notice that the number after the equal sign is the radius after being squared.

ANGLES in CIRCLES

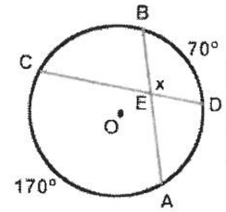
Central Angle:
EQUAL to the arc



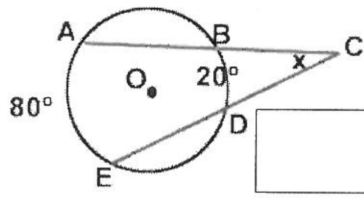
Inscribed Angle:
HALF the arc



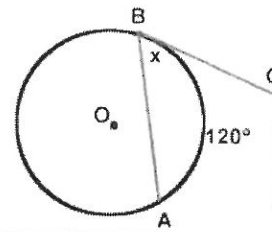
Vertical Angles:
ADD the arcs then divide by 2



Angle OUTSIDE Circle:
SUBTRACT the arcs then divide by 2

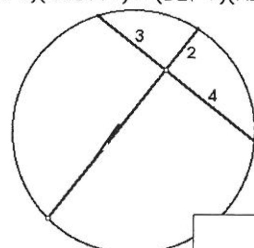


Tangent/Chord Angle:
HALF the arc

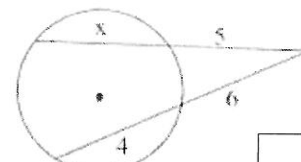


SEGMENTS in CIRCLES

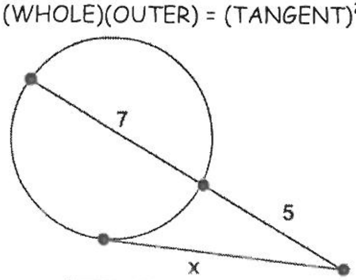
Intersecting Chords:
(LEFT)(RIGHT) = (LEFT)(RIGHT)



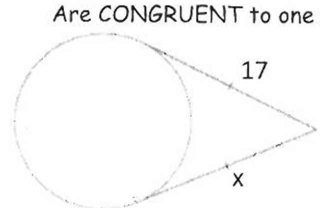
Two Secants:
(WHOLE) (OUTER) = (WHOLE) (OUTER)



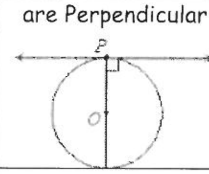
Secant/Tangent:
$(WHOLE)(OUTER) = (TANGENT)^2$



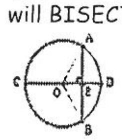
Two Tangents:
Are CONGRUENT to one another



Tangent/Diameter:
are Perpendicular

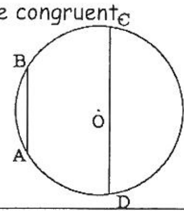


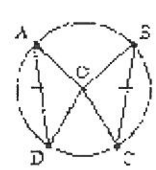
Chord ⊥ Diameter:
will BISECT the chord



If
CD ⊥ AB
Then AE = EB
arc AD = arc DB

Parallel Segments: If 2 segments are parallel, then ARCS BETWEEN are congruent

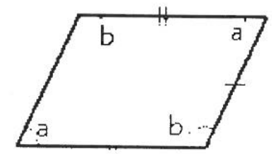




If:
AD = BC
Then
arc AD = arc BC

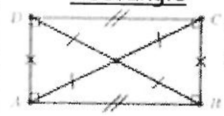
QUADRILATERALS including PARALLELOGRAM FAMILY & TRAPEZOID FAMILY

Parallelogram: opposite sides congruent and parallel
opposite angles congruent
consecutive ∢s supplementary
diagonals BISECT each other



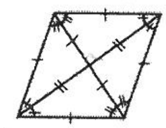
a + b = 180

If ALL Angles EQUAL → **Rectangle:**



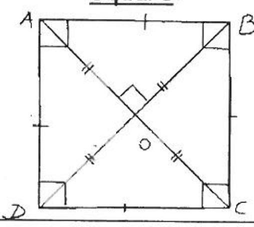
all 90° ∢s
diagonals ≅

If ALL Sides EQUAL → **Rhombus**



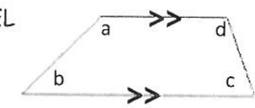
all sides ≅
diagonals ⊥
diagonals BISECT ∢s

Square:



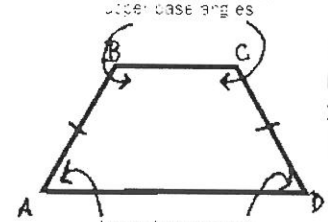
ALL Properties ABOVE

Trapezoid: only ONE pair of opposite sides are PARALLEL
Angles: a + b = 180°, c + d = 180°



if non-parallel sides are CONGRUENT ↓

Isosceles Trapezoid:



Upper Base Angles ≅
Lower Base Angles ≅
1 Upper + 1 Lower = 180°
Diagonals ≅

COORDINATE GEOMETRY PROOFS

Proving a Parallelogram: find DISTANCE of all 4 sides and show opposite sides are CONGRUENT (because they have the same distance).

Proving a Rectangle: find DISTANCE of all 4 sides AND the 2 diagonals and show that opposite sides are CONGRUENT and the diagonals are also.

Proving a Rhombus: find DISTANCE of all 4 sides and show that ALL sides are CONGRUENT (because they have the same distance).

Proving a Square: find DISTANCE of all 4 sides AND the 2 diagonals and show that ALL sides are CONGRUENT and the diagonals are also.

Proving a Trapezoid: find SLOPE of all 4 sides and show that one pair of opposite sides is PARALLEL (b/c they have the same slope) and the other pair is NOT PARALLEL (b/c they have different slopes).

Proving an Isosceles Trapezoid: First, prove it's a trapezoid (see above) then find DISTANCE of the NON-PARALLEL sides and show they are ≅.

So, when do we use the Midpoint Formula in Proofs? Only if we're asked to prove that segments BISECT each other (same midpoint → bisect).

TRIANGLE TYPES

Types of Triangles:

By SIDES → **Scalene**: no ≅ sides
Isosceles: 2 ≅ sides
Equilateral: 3 ≅ sides

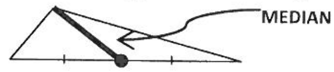
By ANGLES→ **Acute**: all 3 acute ∢s
Right: 1 right ∢ (2 acute)
Obtuse: 1 obtuse (2 acute)

Isosceles Triangle: 2 ≅ sides called LEGS; other side is BASE. Angles opposite legs are ≅ (BASE ANGLES); other angle is VERTEX.

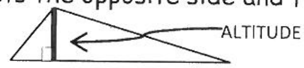
Equilateral Triangle: all sides ≅, all angles ≅ (each angle measures 60°)

SEGMENTS IN TRIANGLES

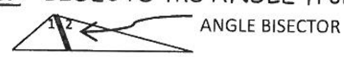
Median: BISECTS the opposite SIDE (intersects at midpoint of opp. side)



Altitude: meets the opposite side and forms a right angle (⊥)



Angle Bisector: BISECTS the ANGLE from where it was drawn



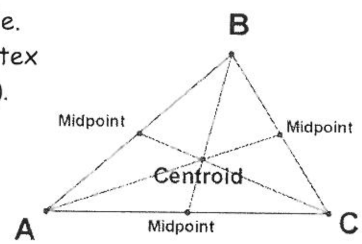
Here: ∢1 ≅ ∢2

Perpendicular Bisector: (1) BISECTS the opposite SIDE and (2) forms a right angle with opposite side (*Notice: It does NOT have to come from opposite ∢*)

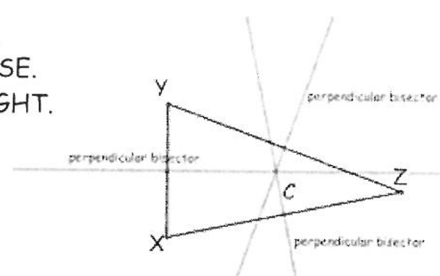
Points of CONCURRENCE: since each triangle has 3 of each of the above line segments, the point where these lines intersect is called...

Name of Point	Intersection of the three...
CENTROID	Medians
CIRCUMCENTER	Perp. Bisectors
INCENTER	Angle Bisectors
ORTHOCENTER	Altitudes

Centroid:
Will always be located inside the triangle.
Divides into **2:1** ratio (section near vertex is twice as long as section near midpt).



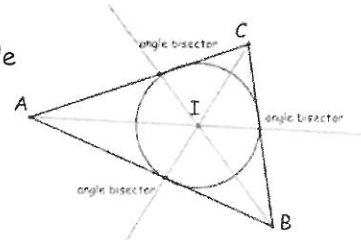
Circumcenter:
Will be inside if triangle is ACUTE.
Will be outside if triangle if OBTUSE.
Will be on triangle if triangle is RIGHT.



SEGMENTS IN TRIANGLES
(continued)

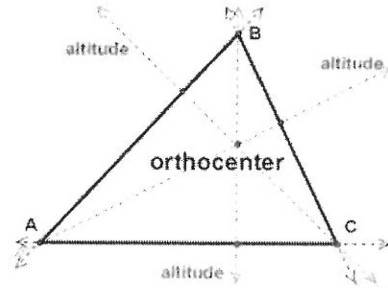
Incenter:

Will always be located inside the triangle.
Incenter will also be the center of the circle inscribed in the triangle.



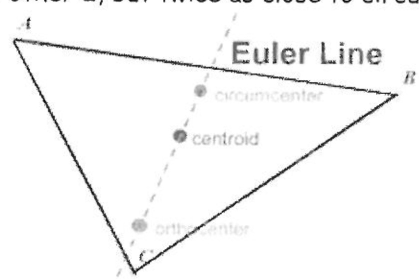
Orthocenter:

Will be inside if triangle is ACUTE.
Will be outside if triangle if OBTUSE.
Will be on triangle if triangle is RIGHT.



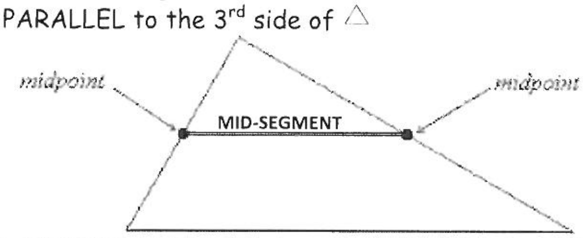
Euler Line:

The three points (**CENTROID, CIRCUMCENTER, ORTHOCENTER**) will always lie on the same line called the Euler Line. The centroid will be between the other 2, but twice as close to circumcenter.



Mid-Segment (Midline): formed when MIDPOINTS of two sides of a triangle are connected. A mid-segment will be...

1. HALF the length of the 3rd side of △
2. PARALLEL to the 3rd side of △

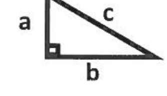


Note: If all three midpoints are connected, a triangle will be formed. This "smaller" triangle will have exactly HALF the perimeter of the big triangle.

RIGHT TRIANGLES

Pythagorean Theorem: used to find the missing side of a right triangle.

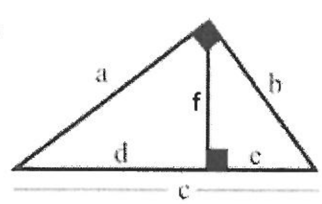
$$a^2 + b^2 = c^2$$



"Altitude drawn to Hypotenuse":

Set up and solve one of these proportions:

$\frac{d}{a} = \frac{a}{c}$ $\frac{e}{b} = \frac{b}{c}$ $\frac{d}{f} = \frac{f}{e}$



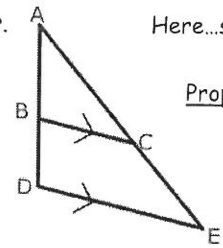
SIMILAR TRIANGLES

Similar Triangles: 2 triangles that have all of their angles congruent and sides are in proportion (**same shape but different size**). The ratio of the sides is the same as the ratio the PERIMETERS, ALTITUDES, MEDIANS, and ANGLE BISECTORS. However, the ratio of the AREAS of the triangles will be the square of the ratio of the sides.

Ex) If the sides of two similar triangles are in the ratio of 2:3, find the ratio of: a) their perimeters

b) their areas

Side-Splitter Theorem: If a segment is parallel to one side of a triangle, then the **sides of the triangle are in proportion** b/c the two triangles are **similar**.



Here...since $\overline{BC} \parallel \overline{DE}$, $\Delta \boldsymbol{ABC} \sim \Delta \boldsymbol{ADE}$, the congruent ∢'s are:

$\measuredangle A \cong \measuredangle A$, $\measuredangle B \cong \measuredangle D$, $\measuredangle C \cong \measuredangle E$

Proportions using SIDES:

$\frac{AB}{BD} = \frac{AC}{CE}$ or $\frac{AB}{AD} = \frac{AC}{AE}$ or $\frac{BD}{AD} = \frac{CE}{AE}$

Proportion using PARALLEL segments: $\frac{AB}{BC} = \frac{AD}{DE}$

Proofs: To prove two triangles are similar, use **AA** (no need to show all 3 angles are ≅). To prove sides are in proportion, first prove Δ's similar by AA then state the proportion by **CSSTP** (Corresponding Sides of Similar Triangles are in Proportion). To prove the product of the line segments are equal, first prove Δ's similar by AA, then state the proportion by CSSTP, then state the product by "**Product of the Means equals Product of the Extremes.**"

CONGRUENT TRIANGLES

To Prove Δ's congruent: use one of the following methods

SSS, SAS, ASA, AAS (also called SAA), **Hyp-Leg**

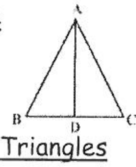
(Never use AAA since that's for similar Δ's nor ASS (SSA) since that's for donkeys)

To Prove Parts (angles or sides) **of Δ's congruent**: First prove Δ's ≅ using one of the above methods, then state the "parts" are congruent by **CPCTC** (Corresponding Parts of Congruent Triangles are Congruent).

Suggestions for Proving Δ's congruent: set up two columns (statements and reasons) and start with the given information marked on the diagram. Use definitions of "vocabulary" words along with theorems as reasons.

Things to Look for if "stuck" on a proof:

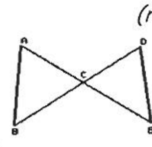
1) Reflexive



Ex) Here... $\overline{AD} \cong \overline{AD}$ (reason: Reflexive Postulate)

2) Isosceles Triangles *Ex) in above Δ, if $\overline{AB} \cong \overline{AC}$, then we can say $\measuredangle B \cong \measuredangle C$ (reason: Base ∢'s in an Isosceles Δ are ≅)*

3) Vertical angles



Ex) Here...$\measuredangle ACB \cong \measuredangle DCE$

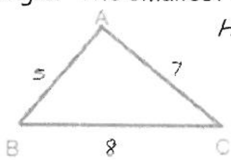
TRIANGLE INEQUALITIES

SIDES of triangles: When we ADD the lengths of the two SMALLEST sides of any triangle, it must be GREATER THAN (and NOT =) the Largest side. *Ex) Can these represent the sides of a triangle?*

a) 3, 5, 7 : YES, b/c $3 + 5 > 7$

b) 2, 4, 2 : NO, b/c $2 + 2 \not> 4$

Largest/Smallest parts of Δ's: The largest side of a Δ is ACROSS from the largest angle. The smallest side of a Δ is ACROSS from the smallest angle. *Ex)*



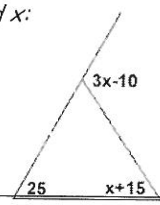
Here...Largest $\measuredangle = \measuredangle A$ (since it's across from 8)

Smallest $\measuredangle = \measuredangle C$ (since it's across from 5)

Exterior $\measuredangle$:

An exterior angle of a triangle will be greater than either of the remote interior angles (interior angles NOT adjacent to it) because its measure is the SUM of these 2 angles.

Ex) Find x:



interior + interior = exterior

$25 + (x+15) = 3x - 10$

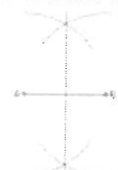
$x + 40 = 3x - 10$

$2x = 50 \Rightarrow$ $x = 25$

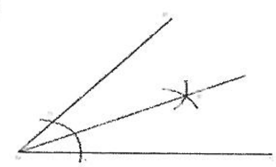
CONSTRUCTIONS

Remember to use a compass and straight edge and to leave all construction markings. Final answers should look like...

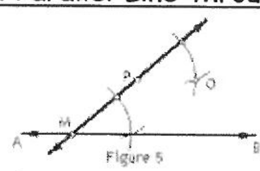
Bisect Segment:

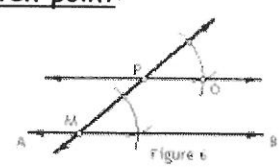


Bisect Angle:

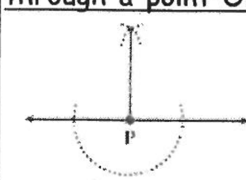


Draw a Parallel Line through a given point:

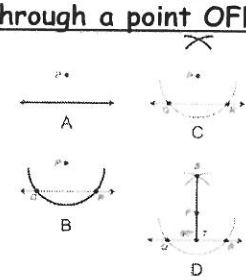




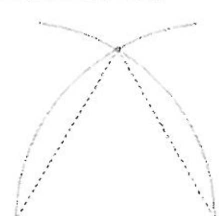
Draw a Perpendicular line through a point ON the line:



Draw a Perpendicular line through a point OFF the line:

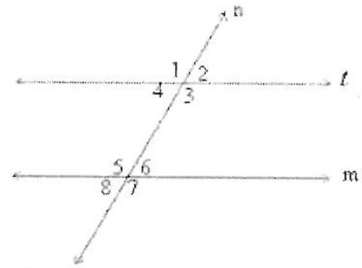


Drawing an Equilateral Triangle:



PARALLEL LINES

When two parallel lines are cut by a transversal, 8 angles are formed (though 4 of the angles will be of one value and 4 will be of another- its supplement).



Alternate Interior Angles: are BETWEEN the parallel lines and on OPPOSITE sides of the transversal (outlining the two angles will form the letter "Z"). *ALTERNATE INTERIOR ANGLES are CONGRUENT*

Ex) $\measuredangle 4 \cong \measuredangle 6 \quad and \quad \measuredangle 3 \cong \measuredangle 5$

Same-Side Interior Angles: are BETWEEN the parallel lines and on SAME sides of the transversal (outlining the two angles will form the letter "C"). *SAME-SIDE INTERIOR ANGLES are SUPPLEMENTARY*

Ex) $\measuredangle 4 + \measuredangle 5 = 180 \quad and \quad \measuredangle 3 + \measuredangle 6 = 180$

Alternate Exterior Angles: are O

Things to Know for the Geometry Regents Exam

ANGLES

Angles INSIDE Triangles: add to 180°

Angles INSIDE Quadrilaterals: add to 360°

Angles INSIDE any polygon with "n" sides: add to $180°(n-2)$

If polygon is regular (all sides and all angles are congruent) then

EACH angle inside the polygon measures: $\frac{180^o(n-2)}{n}$

Angles OUTSIDE any polygon: add to 360^0 where each exterior angle in a *regular* polygon measures $\frac{360}{n}$

Polygons to Know:

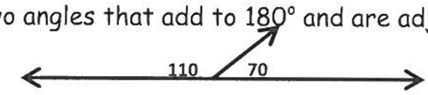
Name	Triangle	Quadrilateral	Pentagon	Hexagon	Octagon	Decagon
# of sides	3	4	5	6	8	10

Complementary Angles: two angles that add to 90°

Supplementary Angles: two angles that add to 180°

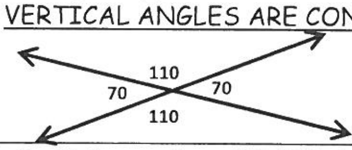
Linear Pair: two angles that add to 180° and are adjacent (form a line)

Ex)



Vertical Angles: (the angles opposite one another that are formed when two lines intersect) VERTICAL ANGLES ARE CONGRUENT.

Ex)



AREA

*Note: When finding the **Volume** of a solid, **"B"** stands for the **Area** of Base.*

Square: $A = s^2$ **Rectangle**: $A = LW$ **Triangle**: $A = \frac{1}{2}bh$

Circle: $A = \pi r^2$ **Trapezoid**: $A = \frac{1}{2}h(b_1 + b_2)$

COORDINATE GEOMETRY including CIRCLES

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula: $M = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of a Circle: where r = radius

centered at origin: $x^2 + y^2 = r^2$

centered at (h, k): $(x - h)^2 + (y - k)^2 = r^2$

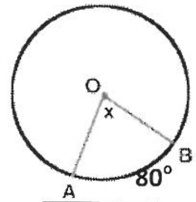
Ex) What is the center and radius of this circle: $(x - 3)^2 + (y + 5)^2 = 16$?

Center: (+3, -5)
Radius: $\sqrt{16} = 4$

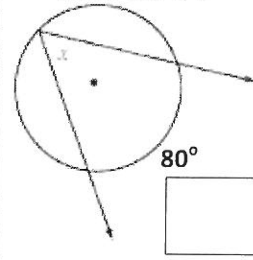
Notice: Change the signs of x and y to find center. If no number is written (as in x^2), then use zero. Also, notice that the number after the equal sign is the radius after being squared.

ANGLES in CIRCLES

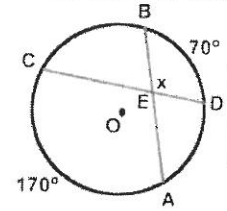
Central Angle:
EQUAL to the arc



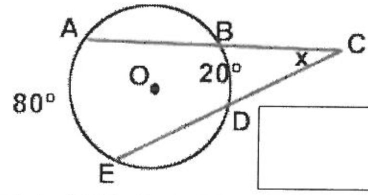
Inscribed Angle:
HALF the arc



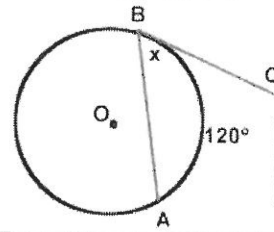
Vertical Angles:
ADD the arcs then divide by 2



Angle OUTSIDE Circle:
SUBTRACT the arcs then divide by 2

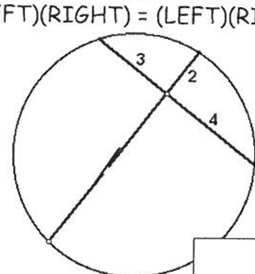


Tangent/Chord Angle:
HALF the arc

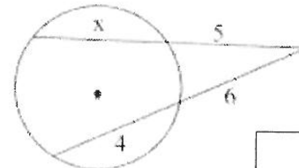


SEGMENTS in CIRCLES

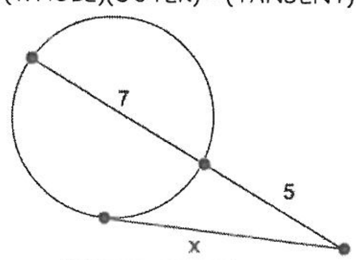
Intersecting Chords:
(LEFT)(RIGHT) = (LEFT)(RIGHT)



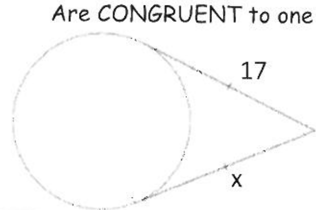
Two Secants:
(WHOLE) (OUTER) = (WHOLE) (OUTER)



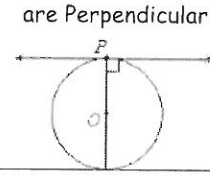
Secant/Tangent:
$(WHOLE)(OUTER) = (TANGENT)^2$



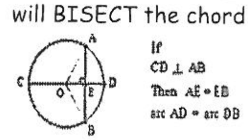
Two Tangents:
Are CONGRUENT to one another



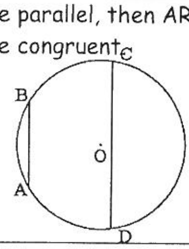
Tangent/Diameter:
are Perpendicular

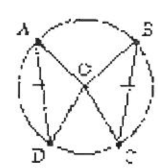


Chord ⊥ Diameter:
will BISECT the chord



Parallel Segments: If 2 segments are parallel, then ARCS BETWEEN are congruent

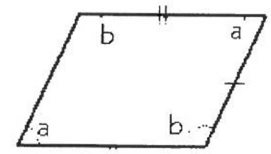




If:
AD = BC
Then
arc AD = arc BC

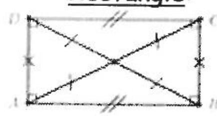
QUADRILATERALS
including PARALLELOGRAM FAMILY & TRAPEZOID FAMILY

Parallelogram: opposite sides congruent and parallel
opposite angles congruent
consecutive ∢s supplementary
diagonals BISECT each other



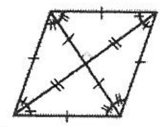
a + b = 180

If ALL Angles EQUAL → **Rectangle**:



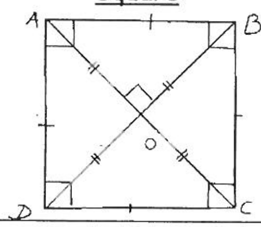
all 90° ∢s
diagonals ≅

If ALL Sides EQUAL → **Rhombus**:



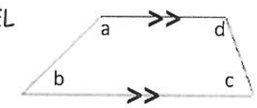
all sides ≅
diagonals ⊥
diagonals BISECT ∢s

Square:



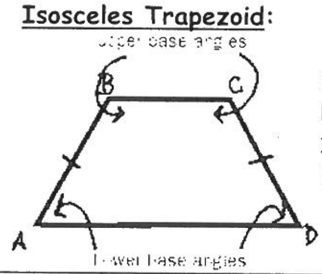
ALL Properties ABOVE

Trapezoid: only ONE pair of opposite sides are PARALLEL
Angles: a + b = 180°, c + d = 180°



if non-parallel sides are CONGRUENT ↓

Isosceles Trapezoid:



Upper Base Angles ≅
Lower Base Angles ≅
1 Upper + 1 Lower = 180°
Diagonals ≅

COORDINATE GEOMETRY PROOFS

Proving a Parallelogram: find DISTANCE of all 4 sides and show opposite sides are CONGRUENT (because they have the same distance).

Proving a Rectangle: find DISTANCE of all 4 sides AND the 2 diagonals and show that opposite sides are CONGRUENT and the diagonals are also.

Proving a Rhombus: find DISTANCE of all 4 sides and show that ALL sides are CONGRUENT (because they have the same distance).

Proving a Square: find DISTANCE of all 4 sides AND the 2 diagonals and show that ALL sides are CONGRUENT and the diagonals are also.

Proving a Trapezoid: find SLOPE of all 4 sides and show that one pair of opposite sides is PARALLEL (b/c they have the same slope) and the other pair is NOT PARALLEL (b/c they have different slopes).

Proving an Isosceles Trapezoid: First, prove it's a trapezoid (see above) then find DISTANCE of the NON-PARALLEL sides and show they are ≅.

So, when do we use the Midpoint Formula in Proofs? Only if we're asked to prove that segments BISECT each other (same midpoint → bisect).

TRIANGLE TYPES

Types of Triangles:

By SIDES → **Scalene**: no ≅ sides

Isosceles: 2 ≅ sides

Equilateral: 3 ≅ sides

By ANGLES→ **Acute**: all 3 acute ∡s

Right: 1 right ∡ (2 acute)

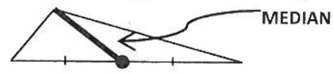
Obtuse: 1 obtuse (2 acute)

Isosceles Triangle: 2 ≅ sides called LEGS; other side is BASE. Angles opposite legs are ≅ (BASE ANGLES); other angle is VERTEX.

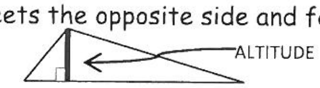
Equilateral Triangle: all sides ≅, all angles ≅ (each angle measures 60°)

SEGMENTS IN TRIANGLES

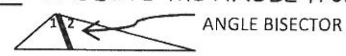
Median: BISECTS the opposite SIDE (intersects at midpoint of opp. side)



Altitude: meets the opposite side and forms a right angle (⊥)

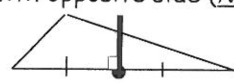


Angle Bisector: BISECTS the ANGLE from where it was drawn



Here: ∡1 ≅ ∡2

Perpendicular Bisector: (1) BISECTS the opposite SIDE and (2) forms a right angle with opposite side (*Notice: It does NOT have to come from opposite ∡*)



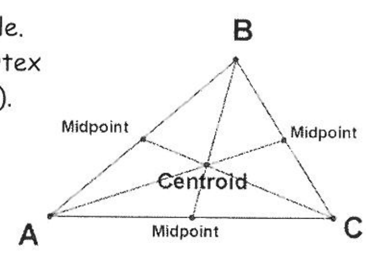
Points of CONCURRENCE: since each triangle has 3 of each of the above line segments, the point where these lines intersect is called...

Name of Point	Intersection of the three...
CENTROID	Medians
CIRCUMCENTER	Perp. Bisectors
INCENTER	Angle Bisectors
ORTHOCENTER	Altitudes

Centroid:

Will always be located inside the triangle.

Divides into **2:1** ratio (section near vertex is twice as long as section near midpt).

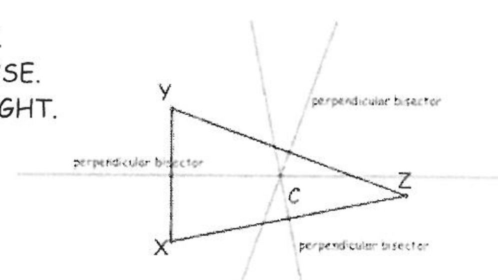


Circumcenter:

Will be inside if triangle is ACUTE.

Will be outside if triangle if OBTUSE.

Will be on triangle if triangle is RIGHT.

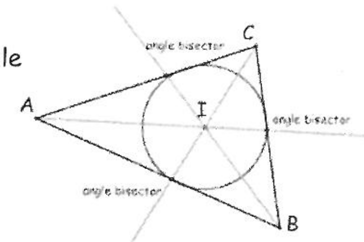


SEGMENTS IN TRIANGLES (continued)

Incenter:

Will always be located <u>inside</u> the triangle.

Incenter will also be the center of the circle inscribed in the triangle.

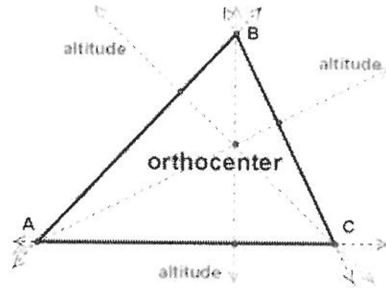


Orthocenter:

Will be <u>inside</u> if triangle is ACUTE.

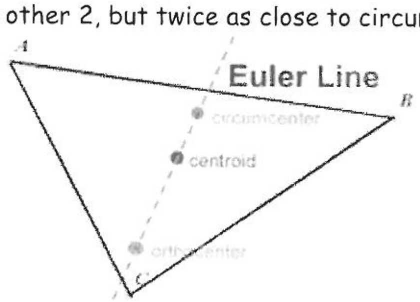
Will be <u>outside</u> if triangle if OBTUSE.

Will be <u>on</u> triangle if triangle is RIGHT.



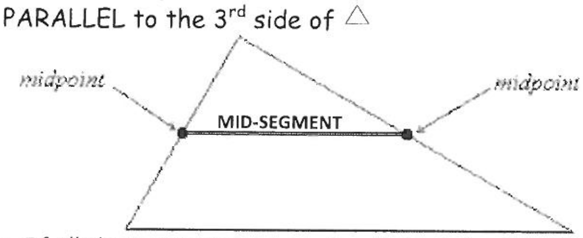
Euler Line:

The three points (**CENTROID, CIRCUMCENTER, ORTHOCENTER**) will always lie on the same line called the Euler Line. The centroid will be between the other 2, but twice as close to circumcenter.



Mid-Segment (Midline): formed when MIDPOINTS of two sides of a triangle are connected. A mid-segment will be...

1. HALF the length of the 3rd side of △
2. PARALLEL to the 3rd side of △

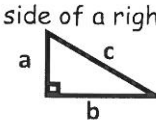


<u>Note</u>: If all three midpoints are connected, a triangle will be formed. This "smaller" triangle will have exactly HALF the perimeter of the big triangle.

RIGHT TRIANGLES

Pythagorean Theorem: used to find the missing side of a right triangle.

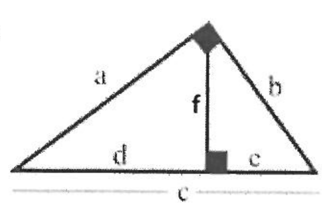
$$a^2 + b^2 = c^2$$



"Altitude drawn to Hypotenuse":

Set up and solve one of these proportions:

$\frac{d}{a} = \frac{a}{c}$ $\frac{e}{b} = \frac{b}{c}$ $\frac{d}{f} = \frac{f}{e}$



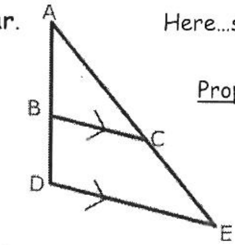
SIMILAR TRIANGLES

Similar Triangles: 2 triangles that have all of their angles congruent and sides are in proportion (**same shape but different size**). The ratio of the sides is the same as the ratio the PERIMETERS, ALTITUDES, MEDIANS, and ANGLE BISECTORS. However, the ratio of the AREAS of the triangles will be the square of the ratio of the sides.

Ex) If the sides of two similar triangles are in the ratio of 2:3, find the ratio of: a) their perimeters ____

b) their areas ____

Side-Splitter Theorem: If a segment is parallel to one side of a triangle, then the **sides of the triangle are in proportion** b/c the two triangles are **similar**.



Here...since $\overline{BC} \parallel \overline{DE}$, $\Delta ABC \sim \Delta ADE$, the congruent ∢'s are:

$∢A \cong ∢A$, $∢B \cong ∢D$, $∢C \cong ∢E$

Proportions using SIDES:

$\frac{AB}{BD} = \frac{AC}{CE}$ or $\frac{AB}{AD} = \frac{AC}{AE}$ or $\frac{BD}{AD} = \frac{CE}{AE}$

Proportion using PARALLEL segments: $\frac{AB}{BC} = \frac{AD}{DE}$

Proofs: To prove two triangles are similar, use **AA** (no need to show all 3 angles are ≅). To prove sides are in proportion, first prove Δ's similar by AA then state the proportion by **CSSTP** (Corresponding Sides of Similar Triangles are in Proportion). To prove the product of the line segments are equal, first prove Δ's similar by AA, then state the proportion by CSSTP, then state the product by "**Product of the Means equals Product of the Extremes.**"

CONGRUENT TRIANGLES

To Prove Δ's congruent: use one of the following methods

SSS, SAS, ASA, AAS (also called SAA), **Hyp-Leg**

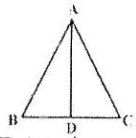
(Never use AAA since that's for similar Δ's nor ASS (SSA) since that's for donkeys)

To Prove Parts (angles or sides) **of Δ's congruent**: First prove Δ's ≅ using one of the above methods, then state the "parts" are congruent by **CPCTC** (Corresponding Parts of Congruent Triangles are Congruent).

Suggestions for Proving Δ's congruent: set up two columns (statements and reasons) and start with the given information marked on the diagram. Use definitions of "vocabulary" words along with theorems as reasons.

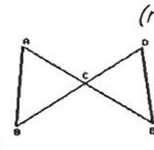
Things to Look for if "stuck" on a proof:

1) Reflexive



Ex) Here... $\overline{AD} \cong \overline{AD}$ (reason: Reflexive Postulate)

2) Isosceles Triangles *Ex) in above Δ, if $\overline{AB} \cong \overline{AC}$, then we can say $∢B \cong ∢C$ (reason: Base ∢'s in an Isosceles Δ are ≅)*



3) Vertical angles *Ex) Here...$∢ACB \cong ∢DCE$*

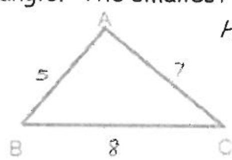
TRIANGLE INEQUALITIES

SIDES of triangles: When we ADD the lengths of the two SMALLEST sides of any triangle, it must be GREATER THAN (and NOT =) the Largest side. *Ex) Can these represent the sides of a triangle?*

a) 3, 5, 7 : YES, b/c $3 + 5 > 7$

b) 2, 4, 2 : NO, b/c $2 + 2 \not> 4$

Largest/Smallest parts of Δ's: The largest side of a Δ is ACROSS from the largest angle. The smallest side of a Δ is ACROSS from the smallest angle. *Ex)*



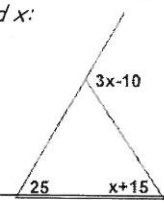
Here...Largest $\measuredangle = \measuredangle A$ (since it's across from 8)

Smallest $\measuredangle = \measuredangle C$ (since it's across from 5)

Exterior $\measuredangle$:

An exterior angle of a triangle will be greater than either of the remote interior angles (interior angles NOT adjacent to it) because its measure is the SUM of these 2 angles.

Ex) Find x:



interior + interior = exterior

$25 + (x+15) = 3x - 10$

$x + 40 = 3x - 10$

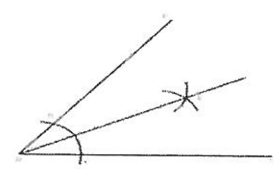
$2x = 50 \Rightarrow$ **$x = 25$**

CONSTRUCTIONS

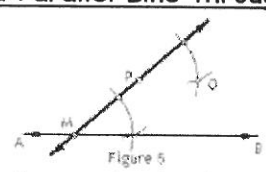
Remember to use a compass and straight edge and to leave all construction markings. Final answers should look like...

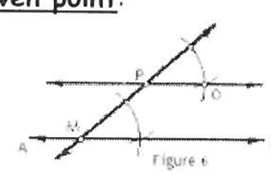
Bisect Segment:

Bisect Angle:

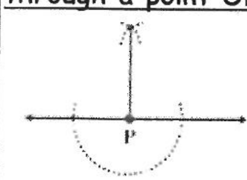


Draw a Parallel Line through a given point:

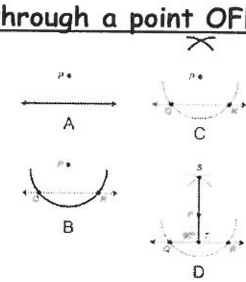




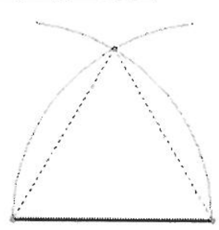
Draw a Perpendicular line through a point ON the line:



Draw a Perpendicular line through a point OFF the line:

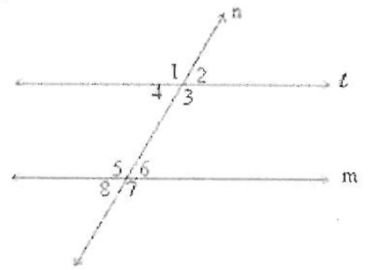


Drawing an Equilateral Triangle:



PARALLEL LINES

When two parallel lines are cut by a transversal, 8 angles are formed (though 4 of the angles will be of one value and 4 will be of another- its supplement).



Alternate Interior Angles: are BETWEEN the parallel lines and on OPPOSITE sides of the transversal (outlining the two angles will form the letter "Z"). *ALTERNATE INTERIOR ANGLES are CONGRUENT*

Ex) $\measuredangle 4 \cong \measuredangle 6$ *and* $\measuredangle 3 \cong \measuredangle 5$

Same-Side Interior Angles: are BETWEEN the parallel lines and on SAME sides of the transversal (outlining the two angles will form the letter "C"). *SAME-SIDE INTERIOR ANGLES are SUPPLEMENTARY*

Ex) $\measuredangle 4 + \measuredangle 5 = 180$ *and* $\measuredangle 3 + \measuredangle 6 = 180$

Alternate Exterior Angles: are OUTSIDE of the parallel lines and on opposite sides of the transversal. *ALTERNATE EXTERIOR ANGLES are CONGRUENT* *Ex)* $\measuredangle 1 \cong \measuredangle 7$ *and* $\measuredangle 2 \cong \measuredangle 8$

Corresponding Angles: are in the same position at each of the 2 points of intersection (outlining the two angles will form the letter "F"). *CORRESPONDING ANGLES are CONGRUENT*

Ex) $\measuredangle 1 \cong \measuredangle 5$, $\measuredangle 2 \cong \measuredangle 6$, $\measuredangle 4 \cong \measuredangle 8$, *and* $\measuredangle 3 \cong \measuredangle 7$

LINES

Equation of a Line: ***y = mx + b*** where *m* = slope and *b* = y-intercept

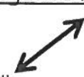
If Vertical line: *x = a number (slope is undefined)* ↕

If Horizontal line: *y = a number (slope is zero)* ↔

Slope of a Line: (using 2 points on the line) $m = \frac{y_2 - y_1}{x_2 - x_1}$

Parallel lines have equal slopes.

Perpendicular Lines have negative reciprocal slopes.

A positive slope looks like…  A negative slope looks like… 

To write the equation of a line: Step 1: Find slope (m). Step 2: Find y- intercept (b) by plugging a point in for (x, y) and slope in for "m" into "y = mx + b" to solve for "b".

Ex) Write the equation of a line perpendicular to y = 2x + 7 that passes through the point (-6,4).

Step 1: since the slope of y = 2x + 7 is "2", the slope of a perp. Line would be $m = -\frac{1}{2}$.

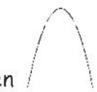
Step 2: [ ] ➡ y = mx + b
$4 = (-\frac{1}{2})(-6) + b$ ➡ [ ]
4 = 3 + b
1 = b

To graph a line on the calculator: Get y alone then type equation into [ ]. For a standard window (between -10 and 10) use [ ] [ ]. Plot at least 3 points that the line passes through from its table of values found under [ ]

PARABOLAS

Equation of a Parabola: $y = ax^2 + bx + c$

If "a" is positive, graph looks like... 
The Vertex is a MINIMUM point

If "a" is negative, then 
The Vertex is a MAXIMUM point

Axis of Symmetry: (the vertical line that passes through vertex)

$$x = \frac{-b}{2a}$$

** This value of x should be in the middle of the table**

Vertex: First find the axis of symmetry using the formula above, then plug that x-value into the parabola's equation to find "y". Vertex = (x, y)

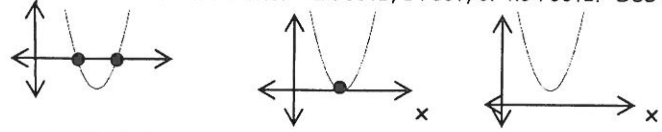
Ex) Find the coordinates of the vertex of the parabola: $y = x^2 - 6x + 4$

Axis of Symmetry: $a = 1$, $b = -6$, $c = 4$ → $x = \frac{-(-6)}{2(1)} = 3$

Vertex: $y = (3)^2 - 6(3) + 4$ → $y = -5$ **VERTEX: (3, -5)**

Roots: The values of *x* where the graph intersects the x-axis (y = 0)

A parabola can have either: 2 roots, 1 root, or no roots. *See diagrams below.*

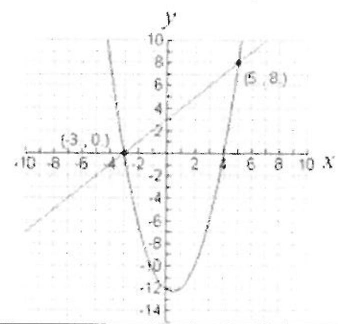


To find the roots algebraically, set equation equal to zero and FACTOR. Set each factor equal to zero and solve for x.

Quadratic/Linear System of Equations:

Two equations will be given. One involving "x^2" (Quadratic) and the other involving "x" (Linear). The solution to the system is the point(s) of intersection of the Parabola and Line. *(Note: there can be 0, 1, or 2)*

Ex)

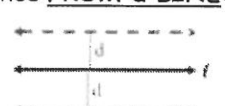


LOCUS

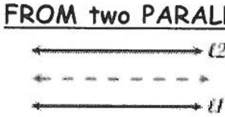
The Five Fundamental Loci: the locus is the set of all points that satisfy a given condition. The locus should be drawn using DOTTED lines.

1. A given distance **FROM a POINT**: Locus is a circle.

2. A given distance **FROM a LINE**: Locus is two parallel lines.

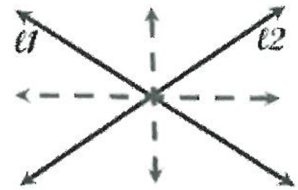


3. Equidistant **FROM two PARALLEL LINES**: Locus is one line.



4. Equidistant **FROM two POINTS**: Locus is one line called the PERPENDICULAR BISECTOR of the segment connecting the points.

5. Equidistant **FROM two INTERSECTING LINES**: Locus is two lines that are the ANGLE BISECTORS of the intersecting lines.



Intersection of Loci: depending on the loci situations described, loci can intersect anywhere between 0 and 4 times. Any example of each is...

0 times:

1 time:

2 times:

3 times:

4 times:

Writing Equation for Loci: since each locus will be a circle, one line or two lines the equations will be...

1. Circle: $(x - h)^2 + (y - k)^2 = r^2$ (see page 1 for more information)
2. Vertical Line: $x = \#$
3. Horizontal Line: $y = \#$
4. All other lines: $y = mx + b$

LOGIC

Logic Symbols:

Symbol	Word(s) it stands for	Vocabulary Word	Truth Value
$\sim$	not	Negation	gives **opposite** truth value
$\wedge$	and	Conjunction	True whenever **both** parts are true
$\vee$	or	Disjunction	True whenever **either** part is true
$\rightarrow$	If...then...	Conditional	Only **False** if T → F
$\leftrightarrow$	if and only if	Biconditional	True whenever both parts have the **same truth value**

Ex) Determine the truth value of the statement:

1) *"If 5 is an odd number, then 17 is not prime."*

TRUE → FALSE = FALSE

2) *"May has exactly 30 days if and only if* February is the first month of the year."

FALSE ↔ FALSE = TRUE

Related Conditionals: For every conditional statement "$p \rightarrow q$", there is...

1. CONVERSE: "Change Order" $(q \rightarrow p)$
2. INVERSE: "I Negate" $(\sim p \rightarrow \sim q)$
3. CONTRAPOSITIVE: "Change Order, Negate Too" $(\sim q \rightarrow \sim p)$
4. LOGICALLY EQUIVALENT: same as the contrapositive. $(\sim q \rightarrow \sim p)$

TRANSFORMATIONS

Isometry: a transformation where the figure remains the SAME SIZE.

Direct Isometry: Isometry where the ORDER of points remains SAME

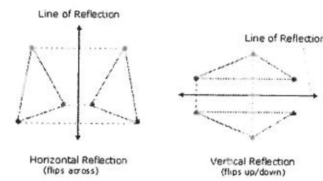
Opposite Isometry: Isometry where the ORDER of points is REVERSED

Line Reflection: a figure FLIPS over a given line

RULES:

- x-axis: (x, y) → (x, -y) "negate the y-value"
- y-axis: (x, y) → (-x, y) "negate the x-value"
- y = x: (x, y) → (y, x) "switch x and y"
- y = -x: (x, y) → (-y, -x) "switch and negate both"

Line Reflections are **OPPOSITE** Isometries

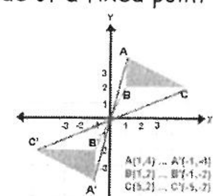


Point Reflection: a figure TURNS 180° to the opposite side of a fixed point

RULE:

origin: (x, y)→ (-x, -y) "negate both x and y"

Point Reflections are **DIRECT** Isometries

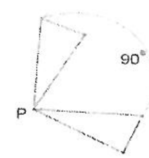


Rotation: a figure TURNS a given # of degrees COUNTERCLOCKWISE

RULES:

- 90°: (x, y) → (-y, x) "switch then negate 1st #"
- 180°: (x, y) → (-x, -y) "negate both x and y"
- 270°: (x, y) → (y, -x) "switch then negate 2nd #"

Rotations are **DIRECT** Isometries

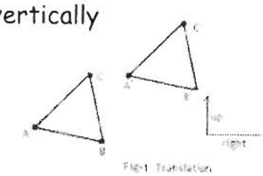


Translation: a figure SLIDES horizontally and/or vertically

RULE:

$T_{a,\,b}$: (x, y) → (x + a, y + b) "ADD"

Translations are **DIRECT** Isometries

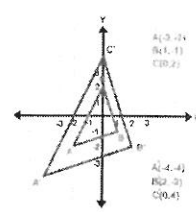


Dilation: a figure ENLARGES/SHRINKS

RULE:

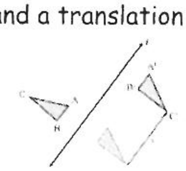
D_k : (x, y) → (kx, ky) "MULTIPLY"

Dilations are **NOT** Isometries



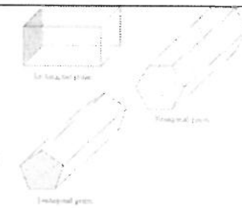
Glide Reflection: the combination of a line reflection and a translation (in either order)

Glide Reflections are **OPPOSITE** Isometries



SOLIDS

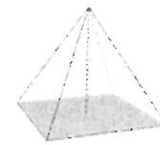
Prism: has two bases that are congruent and parallel. Lateral faces are rectangles.



$V = B \cdot h$

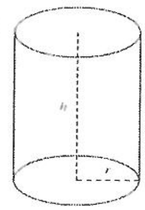
B = AREA of Base
h = height
(see page 1 for Area Formulas)

Pyramid: only has one base. Lateral faces are triangles.



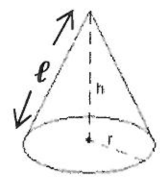
B = AREA of Base
h = height
(see page 1 for Area Formulas)

Cylinder:



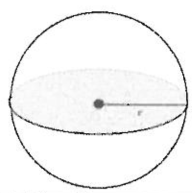
$V = \pi r^2 h$
Surface Area $= 2\pi r^2 + 2\pi r h$
Lateral Area $= 2\pi r h$

Cone:



$V = \frac{1}{3}\pi r^2 h$
Lateral Area $= \pi r \ell$
(ℓ is the slant height)

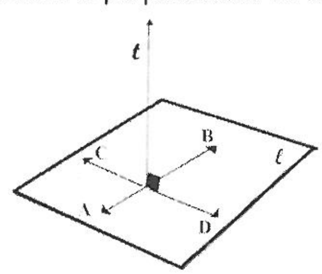
Sphere: the intersection of a plane and a sphere is a circle.



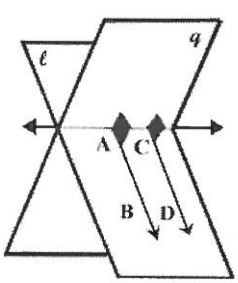
$V = \frac{4}{3}\pi r^3$
Surface Area $= 4\pi r^2$

PLANES

If a line is perpendicular to **each of two intersecting lines** at their point of intersection, then the line is perpendicular to the plane determined by them.



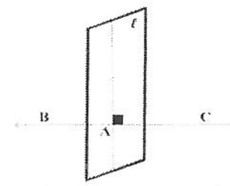
Two lines **perpendicular to the same plane** are COPLANAR.



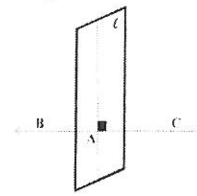
PLANES

(continued)

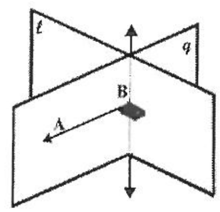
Through a given point, there passes **one and only one plane** perpendicular to a given line.



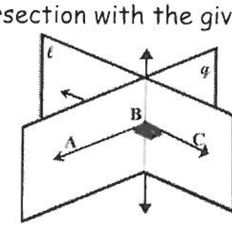
Through a given point, there passes **one and only one line** perpendicular to a given plane.



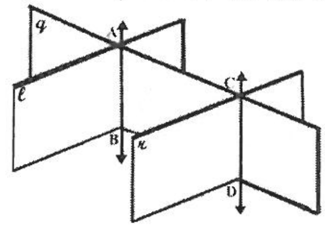
Two planes are perpendicular to each other if and only if **one plane contains a line perpendicular to the second plane.**



If a line is perpendicular to a plane, then any line perpendicular to the given line at its point of intersection with the given plane is **in the given plane**.



If a plane intersects two parallel planes, then the intersection is **two parallel lines.**



If two planes are perpendicular to the same line, then they are parallel.